An Order-Sorted Quantified Modal Logic for Meta-Ontology

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Abstract. The notions of meta-ontology enhance the ability to process knowledge in information systems; in particular, *ontological property classification* deals with the kinds of properties in taxonomic knowledge based on a philosophical analysis. The goal of this paper is to devise a reasoning mechanism to check the *ontological and logical consistency* of knowledge bases, which is important for reasoning services on taxonomic knowledge. We first consider an ontological property classification that is extended to capture individual existence and time and situation dependencies. To incorporate the notion into logical reasoning, we formalize an order-sorted modal logic that involves rigidity, sortality, and three kinds of modal operators (temporal/situational/any world). The sorted expressions and modalities establish axioms with respect to properties, implying the truth of properties in different kinds of possible worlds and in varying domains in Kripke semantics. We provide a prefixed tableau calculus to test the satisfiability of such sorted modal formulas, which validates the ontological axioms of properties.

1 Introduction

Formal ontology deals with the kinds of entities in the real world, such as properties, events, processes, objects, and parts [17]. In this field of research, Guarino and Welty [11] have defined meaningful property classifications as a meta-ontology where properties of individuals are rigorously classified into sortal/non-sortal, rigid/anti-rigid/non-rigid, etc., by a philosophical analysis. The notions of meta-ontology describe the general features of knowledge, which can be applied to enhance knowledge processing in information systems.

On the other hand, order-sorted logic has been recognized as a useful tool for providing logical reasoning systems on taxonomic knowledge [3, 18, 5, 15, 12]. Kaneiwa and Mizoguchi [13] noticed that the ontological property classification [19] fits the formalization of order-sorted logic, and they refined the sorted logic by means of the ontological notion of rigidity and sortality. By using Kripke semantics, *rigid* properties are true in any possible world and *sortal* properties consist of individuals whose parts do not have the same properties. However, they did not cover *individual existence* and *temporal and situational aspects of* *properties* for realistic reasoning services on taxonomic knowledge (only certain temporal aspects were axiomatized by modal and tense operators in [10]).

The first aim of this paper is to present an ontological property classification extended to include the notions of individual existence and time/situation/time-situation dependencies, which are based on the following:

- Entities of properties cannot exist forever in ontological analysis, i.e., every (physical) object will cease to exist at some time.
- One property (e.g., baby) holds depending only on time and is situationally stable, while another (e.g., weapon) holds depending on its use situation and is temporally unstable. For example, a knife can be a weapon in a situation, but it is usually employed as a tool for eating.

These ideas lead to *varying domains*, *times*, and *situations* in possible worlds, which inspire us to define rigidity with individual existence and to further classify anti-rigid properties. In order to model them, we distinguish times and situations from other possible worlds and include varying domains in Kripke semantics.

In order to establish the extensions, we make use of the techniques of quantified modal and temporal logics. Although the logics are usually formalized in *constant domains*, several quantified modal logics address philosophical problems such as *varying domains*, *non-rigid terms*, and local terms. Garson [8] discussed different systems for variants of quantified modal logics. Fitting and Mendelsohn [4] treated rigidity of terms and constant/varying domains by means of a tableau calculus and a predicate abstraction. Meyer and Cerrito [14] proposed a prefixed tableau calculus for all the variants of quantified modal logics with respect to cumulative/varying domains, rigid/non-rigid terms, and local/non-local terms.

Our second aim is to propose an order-sorted modal logic for capturing the extended property classification. This logic requires a combination of order-sorted logic, quantified modal logic, and temporal logic due to their respective features:

1. Order-sorted logic has the advantage that sorted terms and formulas adequately represent properties based on the ontological property classification.
2. Meyer and Cerrito's quantified modal logic provides us with a prefixed tableau calculus for supporting varying domains and non-rigid terms, which can be extended to order-sorted terms/formulas and multi-modal operators.
3. Temporal logic contains temporal representation; however, the standard reasoning systems are propositional [16, 9] or the first-order versions [7, 6, 2] adopt constant domains since they are not easily extended to the first-order temporal logic with varying domains, as discussed in [8].

The proposed logic provides a logical reasoning system for checking the ontological and logical consistency of knowledge bases with respect to properties. Unary predicates and sorts categorized on the basis of rigidity, sortality, and dependencies can be used to represent properties that are appropriately interpreted in modalities and varying domains. We redesign a prefixed tableau calculus for testing the satisfiability of sorted formulas comprising three kinds of modal operators (temporal/situational/any world). This calculus is a variant of Meyer and Cerrito's prefixed tableau calculus that is extended by denoting the kinds of possible worlds in prefixed formulas and by adjusting and supplementing the tableau rules for sorted expressions and multi-modalities supporting individual existence.

2 Property Classification in Semantics

We consider the meaning of properties on the basis of ontological analysis. We begin by characterizing the rigidity of properties in Kripke semantics where a set W of possible worlds w_i is introduced and properties are interpreted differently in each world. Let U be the set of individuals (i.e., the universe), and let $I = \{I_w \mid w \in W\}$ be the set of interpretation functions I_w for all possible worlds $w \in W$. Sortal properties are called *sorts*. We specify that every sort s is interpreted by $I_w(s) \subseteq U$ for each world w, and a subsort relation $s_i < s_j$ is interpreted by $I_w(s_i) \subseteq I_w(s_j)$.

Unlike anti-rigid sorts, substantial sorts (called *types*), constants, and functions are rigid and yield the following semantic constraints. Let τ be a type, c be a constant, and f be a function. For all possible worlds $w_i, w_j \in W$, $I_{w_i}(\tau) = I_{w_j}(\tau)$, $I_{w_i}(c) = I_{w_j}(c)$, and $I_{w_i}(f) = I_{w_j}(f)$ hold. In addition, for each world $w \in W$, every sort s and its sort predicate p_s (as the unary predicate denoted by a sort) are identical in the interpretation and are defined by $I_w(s) = I_w(p_s)$. Standard order-sorted logic does not include the intensional semantics that reflects the rigidity of sorts and sort predicates.

The semantics can be further sophisticated in terms of dependencies of time and situation. As special possible worlds, we exploit *time* and *situation* in order to capture distinctions among anti-rigid sorts (as non-substantial properties). We introduce the set $W_{\mathbf{Tim}}$ of times tm_i and the set $W_{\mathbf{Sit}}$ of situations st_i where $W_{\mathbf{Tim}} \cup W_{\mathbf{Sit}} \subseteq W$. They do not violate rigidity in the interpretation if types, constants, and functions preserve their rigidity in any time and situation. We show dependencies on time and situation that classify anti-rigid sorts as follows:

time dependent: baby, child, youth, adult, elderly
situation dependent: weapon, table, student
time-situation dependent: novice teacher

In Fig.1, these are added to the property classification. The time dependency implies that the truth of a property depends only on time or the meaning of a property is decided essentially by time. For example, the property *baby* is time dependent, so that its entities have the denoting property in a particular time or period.

The situation dependency indicates that the truth of a property is dependent on situation but not on time. Moreover, the situation dependency obtained from extending types (such as *weapon*, *table*, but not *student*) involves a complex idea as mentioned below. We can regard the property *weapon* as a substantial sort (type); however, it is anti-rigid as situation-dependent if it is used as a role expressed by the sort predicate p_{weapon}. E.g., the properties *weapon* and *table* have two kinds of entities: (i) guns and dining tables that innately possess the property of *weapon* and *table* and (ii) knives and boxes that play the roles of *weapon* and *table*. In the latter case, they are not really the aforementioned artifacts and are just referred to as *weapon* and *table*. Thus, knives play the role of a *weapon* only when they are used to attack or kill someone. In the language of order-sorted logic, the former case is an instantiation of a sort (e.g., c_{weapon}), and the latter case is an entity characterized by a sort predicate (e.g., $p_{weapon}(c)$). This consideration disproves the fact that sorts and their sort predicates are interpreted identically in semantics.

Specification 1 *Let τ be a type. If the type predicate p_τ is situation dependent, then $I_w(\tau) \subsetneq I_w(p_\tau)$.*

For example, $gun1 \in I_w(weapon) \cap I_w(p_{weapon})$, whereas $knife1 \notin I_w(weapon)$ and $knife1 \in I_w(p_{weapon})$.

The time-situation dependency is defined such that the truth of a property depends on both time and situation. For example, the property *novice_teacher* is time-situation dependent. Since each novice teacher will become a veteran teacher after a number of years, the property holds only at a particular time under the situation. In other words, the time-situation dependency implies time dependency under a situation, while the situation dependency bears no relationship to time.

We define those dependencies semantically in possible worlds. The basic notion of interpreting time dependency is that for every time-dependent property p and for every individual $d \in U$, if $d \in I_{tm}(p)$ with $tm \in W_{\mathbf{Tim}}$, then another time $tm_j \in W_{\mathbf{Tim}}$ exists such that $d \notin I_{tm_j}(p)$. This is based on the assumption that the same entities (individuals) exist in all possible worlds (called constant domains). However, this assumption does not appear to be realistic since there may be different entities in each possible world. Let U_w be the set of individuals existing in a possible world w. This enables us to consider the case where U_{w_1} and U_{w_2} do not coincide for some possible words $w_1, w_2 \in W$. Consider the following example: every entity of the property $person$ ceases to exist at some time, because no person can live forever. Therefore, we redefine the rigidity of sorts, constants, and functions by supporting individual existence:

Specification 2 (Rigidity) *For all possible worlds $w_i, w_j \in W$, $I_{w_i}(c) = I_{w_j}(c)$ and $I_{w_i}(f) = I_{w_j}(f)$. Let $w \in W$, let $d \in U_w$, and let $R' \subseteq W \times W$ be an accessibility relation. For every type τ, if $d \in I_w(\tau)$ and $(w, w') \in R'$, then $d \in U_{w'}$ implies $d \in I_{w'}(\tau)$. For every anti-rigid sort σ, if $d \in I_w(\sigma)$, then there exists $w_j \in W$ with $(w, w_j) \in R'$ such that $d \notin I_{w_j}(\sigma)$ with $d \in U_{w_j}$.*

By considering the existence of individuals in each world, we specify time/situation/time-situation dependencies where $R_{\mathbf{Tim}} \subseteq W \times W_{\mathbf{Tim}}$ and $R_{\mathbf{Sit}} \subseteq W \times W_{\mathbf{Sit}}$ are accessibility relations from worlds to times and situations, respectively.

Specification 3 (Time Dependency) *Let p be a time-dependent predicate and let $w \in W$.*

1. *(temporally unstable) for all* $(w, tm) \in R_{\mathbf{Tim}}$ *and for all* $d \in U_{tm}$*, if* $d \in I_{tm}(p)$*, then there exists* $tm_j \in W_{\mathbf{Tim}}$ *with* $(tm, tm_j) \in R_{\mathbf{Tim}}$ *such that* $d \notin I_{tm_j}(p)$ *with* $d \in U_{tm_j}$*.*
2. *(situationally stable under time) if* $d \in I_{tm}(p)$ *with* $(w, tm) \in R_{\mathbf{Tim}}$*, then for all situations* $st \in W_{\mathbf{Sit}}$ *with* $(tm, st) \in R_{\mathbf{Sit}}$*,* $d \in U_{st}$ *implies* $d \in I_{st}(p)$*.*

The *temporally unstable* implies that for every time tm accessible from a world w, if an individual d has the property p at tm, we can find a time tm_j accessible from tm where it does not have the property. The *situationally stable under time* defines the fact that for every time tm accessible from a world w, if an individual d has the property p at the time tm, then it has this property in any situation st accessible from the time tm as long as the individual exists. Similar to it, situation dependency can be defined as follows:

Specification 4 (Situation Dependency) *Let p be a situation-dependent predicate and let* $w \in W$*.*

1. *(situationally unstable) for all* $(w, st) \in R_{\mathbf{Sit}}$ *and for all* $d \in U_{st}$*, if* $d \in I_{st}(p)$*, then there exists* $st_j \in W_{\mathbf{Sit}}$ *with* $(st, st_j) \in R_{\mathbf{Sit}}$ *such that* $d \notin I_{st_j}(p)$ *with* $d \in U_{st_j}$*.*
2. *(temporally stable under situation) if* $d \in I_{st}(p)$ *with* $(w, st) \in R_{\mathbf{Sit}}$*, then for all times* $tm \in W_{\mathbf{Tim}}$ *with* $(st, tm) \in R_{\mathbf{Tim}}$*,* $d \in U_{tm}$ *implies* $d \in I_{tm}(p)$*.*

Moreover, we define time-situation dependency as follows:

Specification 5 (Time-Situation Dependency) *Let p be a time-situation dependent predicate and let* $w \in W$*.*

1. *(situationally unstable) the same as in the above.*
2. *(temporally unstable under situation) if* $d \in I_{st}(p)$ *with* $(w, st) \in R_{\mathbf{Sit}}$*, then there are some* $tm_i, tm_j \in W_{\mathbf{Tim}}$ *with* $(st, tm_i), (st, tm_j) \in R_{\mathbf{Tim}}$ *such that* $d \in I_{tm_i}(p)$ *and* $d \notin I_{tm_i}(p)$ *with* $d \in U_{tm_i} \cap U_{tm_j}$*.*

Besides the situational unstability, the *temporally unstable under situation* implies that for every situation st accessible from a world w, if an individual d has the property p in the situation st, then there are times tm_i, tm_j accessible from st such that it has the property p at tm_i, but not at tm_j.

By interpreting such dependencies in possible worlds, the semantics determines ontological differences among anti-rigid properties related to partial rigidity. That is, even if a property is anti-rigid, it may be rigid over a particular kind of possible worlds (e.g., time and situation), as a subset of W. For example, the property *baby* is time dependent, i.e., it is temporally unstable and situationally stable under time. When Bob is a baby, the time dependency derives the fact that Bob is a baby in any situation within the time. Formally, if $bob \in I_{tm}(baby)$, then for any situation st accessible from tm, $bob \in U_{st}$ implies $bob \in I_{st}(baby)$ (if Bob exists in st, he is a baby). It can be viewed as rigid over situations.

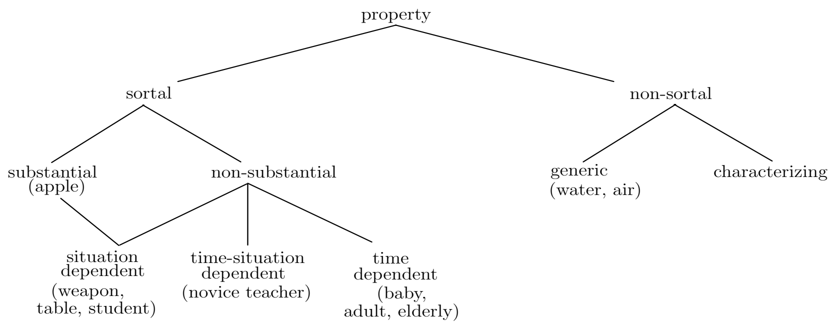

Fig. 1. Ontological property classification

3 Order-Sorted Modal Logic

We define the syntax and semantics of an order-sorted modal logic. The alphabet of a sorted first-order modal language $\mathcal{L}$ with rigidity and sort predicates comprises the following symbols: a countable set T of type symbols (including the greatest type $\top$), a countable set S_A of anti-rigid sort symbols ($T \cap S_A = \emptyset$), a countable set C of constant symbols, a countable set F_n of n-ary function symbols, a countable set P_n of n-ary predicate symbols (including the existential predicate symbol E; the set $P_{T \cup S_A}$ of sort predicate symbols $\{p_s \mid s \in T \cup S_A\}$; and a countable set P_{non} of non-sortal predicate symbols), the connectives $\wedge, \vee, \rightarrow, \neg$, the modal operators $\Box_i, \Diamond_i, \blacksquare, \blacklozenge$, and the auxiliary symbols $($, $)$.

We generally refer to type symbols τ or anti-rigid sort symbols σ as *sort symbols* s. $T \cup S_A$ is the set of sort symbols. V_s denotes an infinite set of variables x_s of sort s. We abbreviate variables $x_\top$ of sort $\top$ as x. The set of variables of all sorts is denoted by $V = \bigcup_{s \in T \cup S_A} V_s$. The unary predicates $p_s \in P_1$ indexed by the sorts s (called *sort predicates*) are introduced for all sorts $s \in T \cup S_A$. In particular, the predicate p_τ indexed by a type τ is called a *type predicate*, and the predicate p_σ indexed by an anti-rigid sort σ is called an *anti-rigid sort predicate*. Hereafter, we assume that the language $\mathcal{L}$ contains all the sort predicates in $P_{T \cup S_A}$. Types can be situation dependent (no type has time/time-situation dependencies), while anti-rigid sorts can be either time, situation, or time-situation dependent (e.g., the type *weapon* is situation dependent, and the anti-rigid sort *adult* is time dependent). Each sort predicate p_s has the same dependency as its sort s.

Definition 1 *A signature of a sorted first-order modal language $\mathcal{L}$ with rigidity and sort predicates (called sorted signature) is a tuple $\Sigma = (T, S_A, \Omega, \leq)$ such that (i) $(T \cup S_A, \leq)$ is a partially ordered set of sorts where $T \cup S_A$ is the union of the set of type symbols and the set of anti-rigid sort symbols in $\mathcal{L}$ and each ordered pair $s_i \leq s_j$ is a subsort relation (implying that s_i is a subsort of s_j) fulfilling the following:*

- *every subsort of types is a sort ($s \leq \tau$) and every subsort of anti-rigid sorts is an anti-rigid sort ($\sigma \leq \sigma'$);*

– *every subsort of time dependent sorts is time or time-situation dependent; every subsort of situation dependent sorts is situation or time-situation dependent; and every subsort of time-situation dependent sorts is time-situation dependent,*

(ii) if $c \in C$, then there is a unique constant declaration $c\colon \ \to \tau \in \Omega$, (iii) if $f \in F_n$ $(n > 0)$, then there is a unique function declaration $f\colon \tau_1 \times \cdots \times \tau_n \to \tau \in \Omega$, and (iv) if $p \in P_n$, then there is a unique predicate declaration $p\colon s_1 \times \cdots \times s_n \in \Omega$ (in particular, if $p_s \in P_{T \cup S_A}$, then there is a unique sort predicate declaration $p_s\colon \tau \in \Omega$ where $s \leq \tau$, and if $p \in P_{non}$, then $p\colon undef \in \Omega$).

A partially ordered set $(T \cup S_A, \leq)$ constructs a sort-hierarchy by suitably ordering different kinds of sorts. A subsort of anti-rigid sorts cannot be a type, a subsort of situation/time-situation dependent sorts cannot be time dependent, and a subsort of time/time-situation dependent sorts cannot be situation dependent. These conditions are guaranteed by the fact that each sort inherits (temporal and situational) unstability and anti-rigidity from its supersorts. For example, the sort *novice_teacher* must be situationally unstable (as time-situation dependent) if the supersort *teacher* is situation dependent.

In sorted signatures, the sorts of constants, functions, and predicates have to be declared by adhering to the rigidity in Specification 2, i.e., since every constant and function is rigid, their sorts have to be rigid. The sort declarations of constants c and functions f are denoted by the forms $c\colon \ \to \tau$ and $f\colon \tau_1 \times \cdots \times \tau_n \to \tau$ where types τ_i, τ are used to declare the sorts. On the other hand, the sort declarations of predicates are denoted by the form $p\colon s_1 \times \cdots \times s_n$ where types and anti-rigid sorts s_i can be used to declare the sorts.

Although the declarations of sort predicates are defined by the greatest sort $\top$ (i.e., $p_s\colon \top$) in [1], we reconsider it in this paper. For each anti-rigid sort σ, there is a basic type τ to be an entity of σ, i.e., every entity of the sort σ must be an entity of the type τ. For example, the anti-rigid sorts *student* and *husband* respectively have the basic types *person* and *male*, defined as the firstness of being able to play the roles. Hence, the declaration of each sort predicate p_s is defined by a type τ such that $s \leq \tau$ (i.e., $p_s\colon \tau$) if it is anti-rigid. Unlike anti-rigid predicates, the declaration of a type predicate $p_{\tau'}$ is simply defined by a necessary condition for the predicate, that is a supersort of the target type (i.e., $p_{\tau'}\colon \tau$ where $\tau' \leq \tau$). For example, the type *person* may have a necessary condition *animal*.

In contrary, each non-sortal property has no such a general type. For instance, a necessary condition of the property *red* appears to be *thing*. However, when considering it as the necessary condition of *red_light*, it is difficult to determine of whether *light* is a thing. Moreover, the property *water* may have the general property *substance*, but it is a non-sortal property (not a type). To avoid such a problem, we express every non-sortal property by a unary predicate (in P_{non}) without a particular sort declaration (denoted instead by $p\colon undef$).

Following the sorted signature, we introduce the three kinds of terms: *typed term*, *anti-rigid sorted term*, and *sorted term* in a sorted first-order modal language $\mathcal{L}_\Sigma$.

Definition 2 *Let $\Sigma = (T, S_A, \Omega, \leq)$ be a sorted signature. The set $\mathcal{T}_\tau^-$ of terms of type τ (called typed terms) is the smallest set such that (i) for every $x_\tau \in V_\tau$, $x_\tau \in \mathcal{T}_\tau^-$, (ii) for every $c \in C$ with $c\colon\ \to \tau \in \Omega$, $c_\tau \in \mathcal{T}_\tau^-$, (iii) if $t_1 \in \mathcal{T}_{\tau_1}^-, \dots, t_n \in \mathcal{T}_{\tau_n}^-$, $f \in F_n$, and $f\colon \tau_1 \times \cdots \times \tau_n \to \tau \in \Omega$, then $f_{\tau^*,\tau}(t_1, \dots, t_n) \in \mathcal{T}_\tau^-$ with $\tau^* = \tau_1, \dots, \tau_n$, and (iv) if $t \in \mathcal{T}_{\tau'}^-$ and $\tau' \leq \tau$, then $t \in \mathcal{T}_\tau^-$. The set $\mathcal{T}_\sigma^-$ of terms of anti-rigid sort σ (called anti-rigid sorted terms) is the smallest set such that (i) for every $x_\sigma \in V_\sigma$, $x_\sigma \in \mathcal{T}_\sigma^-$ and (ii) if $t \in \mathcal{T}_{\sigma'}^-$ and $\sigma' \leq \sigma$, then $t \in \mathcal{T}_\sigma^-$. The set $\mathcal{T}_s$ of terms of sort s (called sorted terms) is the smallest set such that (i) $\mathcal{T}_s^- \subseteq \mathcal{T}_s$ and (ii) if $t \in \mathcal{T}_{s'}$ and $s' \leq s$, then $t \in \mathcal{T}_s$.*

Due to the rigidity of types and anti-rigid sorts, any anti-rigid sorted term (in $\mathcal{T}_\sigma^-$) must be a variable term whereas typed terms (in $\mathcal{T}_\tau^-$) can contain constants and functions. In other words, every anti-rigid sorted term is not rigid (e.g., $x_{student}$) and every typed term is rigid (e.g., c_{person}). We denote $sort(t)$ as the sort of a term t, i.e., $sort(t) = s$ if t is of the form x_s, c_s, or $f_{\tau^*,s}(t_1, \dots, t_n)$. Next, we define sorted modal formulas in the language $\mathcal{L}_\Sigma$.

Definition 3 *The set $\mathcal{F}$ of formulas is the smallest set such that (i) if $t_1 \in \mathcal{T}_{s_1}, \dots, t_n \in \mathcal{T}_{s_n}$, $p \in P_n$, and $p\colon s_1 \times \cdots \times s_n \in \Omega$, then $p(t_1, \dots, t_n)$ is a formula, (ii) if $t \in \mathcal{T}_\tau$, $p \in P_{T \cup S_A}$, and $p_s\colon \tau \in \Omega$, then $p_s(t)$ is a formula, (iii) if $t \in \mathcal{T}_\top$, then $E(t)$ and $p(t)$ are formulas where $p \in P_{non}$, and (iv) if F, F_1, and F_2 are formulas, then $\neg F$, $(\forall x_s)F$, $(\exists x_s)F$, $\Box_i F$, $\Diamond_i F$, $\blacksquare F$, $\blacklozenge F$, $F_1 \wedge F_2$, and $F_1 \vee F_2$ are formulas where $i \in \{\mathbf{Tim}, \mathbf{Sit}\}$.*

The modal formulas are constructed by the modal operators $\blacksquare, \blacklozenge$ (any world), $\Box_{\mathbf{Tim}}$, $\Diamond_{\mathbf{Tim}}$ (temporal), and $\Box_{\mathbf{Sit}}, \Diamond_{\mathbf{Sit}}$ (situational). To axiomatize rigidity and dependencies with individual existence, the modality $\blacksquare F$ and $\Box_i F$ asserts that F holds for any accessible world whenever individuals exist. For example, the sorted modal formula

$$\Box_{\mathbf{Tim}} p_{male}(bob_{person})$$

implies that for any time accessible from a world, Bob is a male person *as long as he exists*. The existential predicate formula $E(t)$ merely asserts that a term t exists. The formula $\neg F_1 \vee F_2$ abbreviates to $F_1 \to F_2$.

We define the semantics for a sorted first-order modal language $\mathcal{L}_\Sigma$. A sorted Σ-structure M is a tuple (W, w_0, R, R', U, I) such that (i) W is a superset of $\bigcup_{1 \leq i \leq n} W_i$ where W_i is a non-empty set of worlds and $W_i \cap W_j = \emptyset$ $(i \neq j)$; (ii) $R = (R_1, \dots, R_n)$ where R_i is a subset of $W \times W_i$; (iii) R' is a superset of $R_1 \cup \cdots \cup R_n$; (iv) U is a superset of $\bigcup_{w \in W} U_w$ where U_w is the set of individuals in world w[3]; and (v) $I = \{I_w \mid w \in W\}$ is the set of interpretation functions I_w for all worlds $w \in W$ with the following conditions:

1. if $s \in T \cup S_A$, then $I_w(s) \subseteq U_w$ (in particular, $I_w(\top) = U_w$). In addition, $I(s)$ is a superset of $\bigcup_{w \in W} I_w(s)$ such that $U_w \cap I(s) \subseteq I_w(s)$[4],

[3] Each world can have a different domain (possibly $U_{w_1} \neq U_{w_2}$).

[4] If an individual in $I(s)$ exists in a world w, then it must belong to the interpretation $I_w(s)$ in w. That is, $I(s)$ may be constructed by $\bigcup_{w \in W} I_w(s)$ and individuals non-existing in any world.

2. if $s_i \leq s_j$ with $s_i, s_j \in T \cup S_A$, then $I_w(s_i) \subseteq I_w(s_j)$,
3. if $c \in C$ and $c\colon \to \tau \in \Omega$, then $I_w(c) \in I(\tau)$,
4. if $f \in F_n$ and $f\colon \tau_1 \times \cdots \times \tau_n \to \tau \in \Omega$, then $I_w(f)\colon I(\tau_1) \times \cdots \times I(\tau_n) \to I(\tau)$,
5. if $p \in P_n$ and $p\colon s_1 \times \cdots \times s_n \in \Omega$, then $I_w(p) \subseteq I_w(s_1) \times \cdots \times I_w(s_n)$ (in particular, if $p_s \in P_{T \cup S_A}$ and $p_s\colon \tau \in \Omega$, then $I_w(p_s) \subseteq I_w(\tau)$),
6. if $p \in P_{non}$ and $p\colon undef \in \Omega$, then $I_w(p) \subseteq U_w \cup \Delta_w$ where Δ_w is an uncountably infinite set.

The semantic difference between types τ and anti-rigid sorts σ (i.e., rigidity) is characterized in the following definition using Specification 2. By restricting sorted Σ-structures in terms of rigidity and time and situation dependencies (introducing accessibility relations from worlds to times and situations), we obtain a class of sorted Σ-structures as follows:

Definition 4 *A sorted Σ-structure with rigidity and time/situation dependencies (called sorted Σ^+-structure) is a sorted Σ-structure $M = (W, w_0, R, R', U, I)$ such that*
(rigidity)
1. *$R' \supseteq R_{\mathbf{Tim}} \cup R_{\mathbf{Sit}}$ is reflexive and transitive,*
2. *Specification 2 (in Section 2),*
3. *for every generic predicate[5] $p \in P_{non}$, if $d \in I_w(p)$ and $(w, w') \in R'$, then $d \in U_{w'} \cup \Delta_{w'}$ implies $d \in I_{w'}(p)$,*

(time and situation dependencies)
4. *W is a superset of $W_{\mathbf{Tim}} \cup W_{\mathbf{Sit}}$ where $W_{\mathbf{Tim}}$ is the set of times and $W_{\mathbf{Sit}}$ is the set of situations ($W_{\mathbf{Tim}} \cap W_{\mathbf{Sit}} = \emptyset$),*
5. *$R = (R_{\mathbf{Tim}}, R_{\mathbf{Sit}})$ where $R_{\mathbf{Tim}} \subseteq W \times W_{\mathbf{Tim}}$ is reflexive and transitive over $W_{\mathbf{Tim}} \times W_{\mathbf{Tim}}$ and $R_{\mathbf{Sit}} \subseteq W \times W_{\mathbf{Sit}}$ is reflexive and transitive over $W_{\mathbf{Sit}} \times W_{\mathbf{Sit}}$,*
6. *Specifications 3-5 (in Section 2).*

Further, there exist the correspondences between sorts and their sort predicates (based on Specification 1). If a type τ is situation dependent, then it and its type predicate are *extensible* with $I_w(\tau) \subsetneq I_w(p_\tau)$ (in addition, $I_w(p_\tau) \subseteq I_w(p_{\tau'})$ if $\tau \leq \tau'$ and τ, τ' are extensible). Any other sort and its sort predicate are *inextensible* with $I_w(s) = I_w(p_s)$. For every extensible type predicate p_τ, we assume that there exists an anti-rigid sort σ as the role of type τ with $\sigma \leq \tau$ such that $I_w(\sigma) = I_w(p_\tau) \backslash I_w(\tau)$. E.g., the anti-rigid sort *temporary_weapon* is the role of the type *weapon*.

To define satisfiability of formulas, we employ the existence of terms in each world. Let $M = (W, w_0, R, R', U, I)$ be a sorted Σ^+-structure, let $w \in W$, and let $[\![t]\!]_w$ be the denotation of a term t in w. The set Nex_w of formulas with terms non-existing in w is the smallest set such that (i) $p(t_1, \ldots, t_n) \in Nex_w$ *iff* for some ground term $t \in \{t_1, \ldots, t_n\}$, $[\![t]\!]_w \notin U_w$; (ii) $\neg F$, $(\forall x_s)F$, $(\exists x_s)F \in Nex_w$ *iff* $F \in Nex_w$; (iii) $\Box_i F$, $\Diamond_i F$, $\blacksquare F$, $\blacklozenge F \notin Nex_w$; (iv) $F_1 \wedge F_2 \in Nex_w$ *iff* $F_1 \in Nex_w$ or $F_2 \in Nex_w$; and (v) $F_1 \vee F_2 \in Nex_w$ *iff* $F_1 \in Nex_w$ and $F_2 \in Nex_w$. This set is important for the interpretation of modality. In Definition 5, the modal formula $\blacksquare F$ is satisfied in a world w if for any world w' accessible from w, F is satisfied in w' ($w' \models F$) or some ground terms in F do not exist in w' ($F \in Nex_{w'}$).

[5] A non-sortal predicate is called generic if it is rigid.

Definition 5 *The Σ^+-satisfiability relation $w \models F$ is defined inductively as follows:*

1. $w \models p(t_1, \ldots, t_n)$ *iff* $([\![t_1]\!]_w, \ldots, [\![t_n]\!]_w) \in I_w(p)$.
2. $w \models E(t)$ *iff there exists* $d \in U_w$ *such that* $[\![t]\!]_w = d$.
3. $w \models (\forall x_s)F$ *iff for all* $d \in I_w(s)$, $w \models F[x_s/\bar{d}]$.
4. $w \models (\exists x_s)F$ *iff for some* $d \in I_w(s)$, $w \models F[x_s/\bar{d}]$.
5. $w \models \Box_i F$ *(resp.* $\blacksquare F$*) iff for all* $w' \in W_i$ *with* $(w, w') \in R_i$ *(resp.* R'*)* , $w' \models F$ *or* $F \in Nex_{w'}$.
6. $w \models \Diamond_i F$ *(resp.* $\blacklozenge F$*) iff for some* $w' \in W_i$ *with* $(w, w') \in R_i$ *(resp.* R'*)*, $w' \models F$ *and* $F \notin Nex_{w'}$.

The formulas $\neg F$, $F_1 \wedge F_2$, and $F_1 \vee F_2$ are satisfied in the usual manner of first-order logic. Let F be a formula. It is Σ^+-true in M if $w_0 \models F$ (M is a Σ^+-model of F). If F has a Σ^+-model, it is Σ^+-satisfiable, otherwise, it is Σ^+-unsatisfiable. F is Σ^+-valid if every sorted Σ^+-structure is a Σ^+-model of F.

Proposition 1 *Let p be an inextensible type predicate $p_{\tau'}$ with $p_{\tau'} : \tau \in \Omega$ or generic non-sortal predicate (in this case, $\tau = \top$), and p_σ be an anti-rigid sort predicate with $p_\sigma : \tau \in \Omega$. The following axioms are Σ^+-valid.*

1. **Rigid predicate axiom:**
   $(\forall x_\tau)(p(x_\tau) \rightarrow \blacksquare p(x_\tau))$
2. **Anti-rigid predicate axiom:**
   $(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \blacklozenge(\neg p_\sigma(x_\tau)))$
3. **Time dependency axiom:**
   $\Box_{\mathbf{Tim}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \Diamond_{\mathbf{Tim}}(\neg p_\sigma(x_\tau)))$
   $\Box_{\mathbf{Tim}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \Box_{\mathbf{Sit}} p_\sigma(x_\tau))$
4. **Situation dependency axiom:**
   $\Box_{\mathbf{Sit}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \Diamond_{\mathbf{Sit}}(\neg p_\sigma(x_\tau)))$
   $\Box_{\mathbf{Sit}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \Box_{\mathbf{Tim}} p_\sigma(x_\tau))$
5. **Time-situation dependency axiom:**
   $\Box_{\mathbf{Sit}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \Diamond_{\mathbf{Sit}}(\neg p_\sigma(x_\tau)))$
   $\Box_{\mathbf{Sit}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow (\Diamond_{\mathbf{Tim}} p_\sigma(x_\tau) \wedge \Diamond_{\mathbf{Tim}}(\neg p_\sigma(x_\tau))))$

These axioms reflect the intension of rigidity and time and situation dependencies. We denote the set of axioms by $\mathcal{A}_{\mathcal{L}_\Sigma}$.

Example 1 Let p_{apple} be an inextensible type predicate and $p_{nov\_teacher}$ be an anti-rigid sort predicate (time-situation dependent) where $p_{apple} : \mathit{fruit}$ and $p_{nov\_teacher} : \mathit{person}$ in Ω. Then, the two sorted modal formulas $p_{apple}(c_{\mathit{fruit}}) \rightarrow \blacksquare p_{apple}(c_{\mathit{fruit}})$ and

$$\Box_{\mathbf{Sit}}(p_{nov\_teacher}(john_{person}) \rightarrow (\Diamond_{\mathbf{Tim}} p_{nov\_teacher}(john_{person}) \wedge \Diamond_{\mathbf{Tim}} \neg p_{nov\_teacher}(john_{person})))$$

are Σ^+-valid. The subformula $\blacksquare p_{apple}(c_{\mathit{fruit}})$ expresses rigidity with *individual existence*. This implies "c_{fruit} is an apple in any world as long as it exists," (for any accessible world $w \in W$, if $[\![c_{fruit}]\!]_w \in U_w$ (it exists), then $[\![c_{fruit}]\!]_w \in I_w(p_{apple})$), but this does not imply "c_{fruit} is an apple forever" or "c_{fruit} exists forever." Moreover, the subformula $\Diamond_{\mathbf{Tim}} \neg p_{nov\_teacher}(john_{person})$ indicates "there is a time where $john_{person}$ exists but is not a novice teacher."

4 Tableau Calculus

We present a prefixed tableau calculus for the order-sorted modal logic. Let A be a closed formula in negation normal form (i.e., negation occurs only in front of an atomic formula) and S be a finite set of closed formulas in negation normal form. We define S^n as the set, A^n as the formula, and t^n as the term obtained from S, A, and t by annotating each non-annotated constant and function with level n ($\in \mathbb{N}$), such that (i) if $S = \{A_1, \dots, A_m\}$ then $S^n = \{A_1^n, \dots, A_m^n\}$, (ii) if $A_i = p(t_1, \dots, t_m)$ then $A_i^n = p(t_1^n, \dots, t_m^n)$, (iii) if $A_i \neq p(t_1, \dots, t_m)$ then $A_i^n = A_i$, (iv) if $t = x_s$ then $t^n = x_s$, (v) if $t = c_\tau$ then $t^n = c_\tau^n$, and (vi) if $t = f_{\tau^*,\tau}(t_1, \dots, t_m)$ then $t^n = f^n_{\tau^*,\tau}(t_1^n, \dots, t_m^n)$. The annotated term t^n syntactically implies that it exists in the world corresponding to level n. Each node in a tableau is labeled with a prefixed formula set $(i, n)\colon S$ where $i \in \{W, Tim, Sit\}$ and $n \in \mathbb{N}$. The initial tableau for S is the single node $(W, 1)\colon (S^+)^1$ where S^+ is the smallest superset of S obtained by adding $\blacksquare F$ for all axioms F in $\mathcal{A}_{\mathcal{L}_\Sigma}$. The initial tableau $(W, 1)\colon (S^+)^1$ plays the key role in deciding Σ^+-satisfiability for S since it includes the formulas $\blacksquare F$ for all axioms F in $\mathcal{A}_{\mathcal{L}_\Sigma}$. The axioms characterize the meta-features of properties, and the attached operator $\blacksquare F$ validates the axioms in any world by applications of π_j-/$\pi_{i \mapsto j}$-/π_W-rules.

We introduce a set of tableau rules as follows: A ground term t is with level n if the annotated term t^n occurs in an ancestor. Let $i \in \{W,\ Tim,\ Sit\}$, $j \in \{Tim,\ Sit\}$, let t be any ground term with level n, and let comma be the union of sets (i.e., $S_1, S_2 = S_1 \cup S_2$, $A, S = \{A\} \cup S$, and $A, B = \{A\} \cup \{B\}$).

Conjunction and disjunction rules
$\dfrac{(i,n)\colon A \wedge B, S}{(i,n)\colon A^n, B^n, S}\ (\alpha) \qquad \dfrac{(i,n)\colon A \vee B, S}{(i,n)\colon A^n, S \quad (i,n)\colon B^n, S}\ (\beta)$

In α-rule and β-rule, the decomposed formulas A and B are annotated with level n (such as A^n and B^n) since they may be atomic formulas. For example, if $p(t) \wedge F$ is decomposed to $p(t)$ and F by α-rule, then we obtain the annotated atomic formula $p(t^n)$.

Existential predicate rules
$\dfrac{(i,n)\colon \neg E(t), S}{(i,n)\colon \bot, \neg E(t), S}\ (E) \qquad \dfrac{(i,n)\colon S}{(i,n)\colon E(a_n^n), p_s(a_n^n), S}\ (I)$

In I-rule, a_n is the dummy constant for level n such that $sort(a_n) \leq s$ for all sorts $s \in T \cup S_A$ (a_n^n is the annotated term of a_n with level n). By an application of I-rule, a_n is introduced as a ground term with level n. The dummy constant for each level is used to guarantee the non-empty domain of each world.

In the modal operation rules, $*S$ denotes $\{*F \mid F \in S\}$ for $* \in \{\blacksquare, \Box_i\}$ (possibly $*S = \emptyset$) and $\langle *_1, *_2 \rangle S$ denotes $*_1 S \cup *_2 S$ (i.e., $\langle \Box_i, \blacksquare \rangle S = \Box_i S \cup \blacksquare S$). Let $\mathcal{T}_0$ be the set of ground terms. The translation $\mathcal{E}\colon \mathcal{F} \to \mathcal{F}$ is defined as follows: (i) $\mathcal{E}(p(t_1, \ldots, t_n)) = \emptyset$ if $\{t_1, \ldots, t_n\} \not\subseteq \mathcal{T}_0$, otherwise $\mathcal{E}(p(t_1, \ldots, t_n)) = \bigwedge_{t \in \{t_1, \ldots, t_n\} \cap \mathcal{T}_0} E(t)$; (ii) $\mathcal{E}(*F) = \mathcal{E}(F)$ for every $* \in \{\neg, \forall x_s, \exists x_s\}$; (iii) $\mathcal{E}(*F) = \emptyset$ for every $* \in \{\Box_i, \Diamond_i, \blacksquare, \blacklozenge\}$; (iv) $\mathcal{E}(F_1 \wedge F_2) = \mathcal{E}(F_1) \wedge \mathcal{E}(F_2)$; and (v) $\mathcal{E}(F_1 \vee F_2) = \mathcal{E}(F_1) \vee \mathcal{E}(F_2)$. Moreover, we define $S \vee \neg\mathcal{E}(S) = \{F \vee \neg\mathcal{E}(F) \mid F \in S\}$.

Modal operator rules

$$\frac{(j,n)\colon \Box_j A, S}{(j,n)\colon A \vee \neg\mathcal{E}(A), \Box_j A, S}\ (\nu_j) \qquad \frac{(j,n)\colon \Diamond_j A, \langle \Box_j, \blacksquare \rangle S, S'}{(j,n{+}1)\colon A \wedge \mathcal{E}(A), S \vee \neg\mathcal{E}(S), \langle \Box_j, \blacksquare \rangle S}\ (\pi_j)$$

$$\frac{(i,n)\colon \Diamond_j A, \langle \Box_j, \blacksquare \rangle S, S'}{(j,n{+}1)\colon A \wedge \mathcal{E}(A), S \vee \neg\mathcal{E}(S), \blacksquare S}\ (\pi_{i \mapsto j}) \qquad \frac{(i,n)\colon \blacklozenge A, \blacksquare S, S'}{(W,n{+}1)\colon A \wedge \mathcal{E}(A), S \vee \neg\mathcal{E}(S), \blacksquare S}\ (\pi_W)$$

$$\frac{(i,n)\colon \blacksquare A, S}{(i,n)\colon A \vee \neg\mathcal{E}(A), \Box_{\mathbf{Tim}} A, \Box_{\mathbf{Sit}} A, \blacksquare A, S}\ (\blacksquare\Box) \qquad \frac{(i,n)\colon \Diamond_j A, S}{(i,n)\colon \blacklozenge A, \Diamond_j A, S}\ (\Diamond\blacklozenge)$$

In $\pi_{i \mapsto j}$-rule, $i \neq j$, in π_j-/$\pi_{i \mapsto j}$-rules, S' is a set of closed formulas without the forms $\blacksquare F$ and $\Box_j F$, and in π_W-rule, S' is a set of closed formulas without the form $\blacksquare F$.

Sorted quantifier rules

$$\frac{(i,n)\colon \forall x_\tau A, S}{(i,n)\colon A[x_\tau/t]^n, \forall x_\tau A, S}\ (\gamma_\tau) \qquad \frac{(i,n)\colon p_{s'}(t^n), \forall x_s A, S}{(i,n)\colon p_{s'}(t^n), A[x_s/t]^n, \forall x_s A, S}\ (\gamma_s)$$

$$\frac{(i,n)\colon \exists x_\tau A, S}{(i,n)\colon E(c_\tau^n), A[x_\tau/c_\tau]^n, \exists x_\tau A, S}\ (\delta_\tau) \qquad \frac{(i,n)\colon \exists x_\sigma A, S}{(i,n)\colon p_\sigma(c_\tau^n), A[x_\sigma/c_\tau]^n, \exists x_\sigma A, S}\ (\delta_\sigma)$$

In γ_τ-rule, $sort(t) \leq \tau$, in γ_s-rule, $s' \leq s$ and if s is extensible, then $p_{s'}$ is an anti-rigid sort predicate, in δ_τ-rule, c_τ is a constant not in $\{\exists x_\tau A\} \cup S$, and in δ_σ-rule, c_τ is a constant not in $\{\exists x_\sigma A\} \cup S$ where $p_\sigma\colon \tau \in \Omega$.

Sort predicate rules

$$\frac{(i,n)\colon S}{(i,n)\colon p_\tau(t^n), S}\ (p_\tau) \qquad \frac{(i,n)\colon p_s(t^n), S}{(i,n)\colon p_{s'}(t^n), p_s(t^n), S}\ (<)$$

In p_τ-rule, $sort(t) \leq \tau$, and in $<$-rule, $s < s'$.

A tableau rule is called *static* if it does not change the level n (i.e., $(i,n)\colon S$ is expanded to $(i,n)\colon S'$ by an application of the rule), it is called *dynamic* otherwise (e.g., π_j-/$\pi_{i \mapsto j}$-/π_W-rules are dynamic). The set of closed nodes in a tableau for $(i,n)\colon S$ is defined as follows: (i) if a node contains two complementary literals ($\neg A$ and A^n) or the clash symbol $\perp$, then it is closed and (ii) if all the children of a node are closed, then it is closed. A tableau is closed if the root is closed.

Theorem 1 (Completeness) *There exists a closed tableau for S if and only if S is Σ^+-unsatisfiable.*

$$
\begin{array}{cl}
(W,1)\colon p_{apple}(c_{fruit}) \land \blacklozenge(\neg p_{apple}(c_{fruit})), \blacksquare\forall x_{fruit}(p_{apple}(x_{fruit}) \to \blacksquare p_{apple}(x_{fruit})) & \\ \hline
(W,1)\colon p_{apple}(c_{fruit}) \land \blacklozenge(\neg p_{apple}(c_{fruit})), \forall x_{fruit}(p_{apple}(x_{fruit}) \to \blacksquare p_{apple}(x_{fruit})) & (\blacksquare\Box) \\ \hline
(W,1)\colon p_{apple}(c^1_{fruit}), \blacklozenge(\neg p_{apple}(c_{fruit})), \forall x_{fruit}(p_{apple}(x_{fruit}) \to \blacksquare p_{apple}(x_{fruit})) & (\alpha) \\ \hline
(W,1)\colon p_{apple}(c^1_{fruit}), \blacklozenge(\neg p_{apple}(c_{fruit})), p_{apple}(c_{fruit}) \to \blacksquare p_{apple}(c_{fruit}) & (\gamma_\tau) \\ \hline
\begin{array}[t]{ll}
(W,1)\colon p_{apple}(c^1_{fruit}), \neg p_{apple}(c_{fruit}) &
\begin{array}[t]{cl}
(W,1)\colon p_{apple}(c^1_{fruit}), \blacklozenge(\neg p_{apple}(c_{fruit})), \blacksquare p_{apple}(c_{fruit}) & \\ \hline
(W,2)\colon \neg p_{apple}(c_{fruit}) \land E(c_{fruit}), p_{apple}(c_{fruit}) \lor \neg E(c_{fruit}) & (\pi_W) \\ \hline
(W,2)\colon \neg p_{apple}(c_{fruit}), E(c^2_{fruit}), p_{apple}(c_{fruit}) \lor \neg E(c_{fruit}) & (\alpha) \\ \hline
\begin{array}[t]{ll}
(W,2)\colon \neg p_{apple}(c_{fruit}), p_{apple}(c^2_{fruit}) &
\begin{array}[t]{cl}
(W,2)\colon \neg E(c_{fruit}) & \\ \hline
(W,2)\colon \bot & (E)
\end{array}
\end{array} & (\beta)
\end{array}
\end{array} & (\beta)
\end{array}
$$

Fig. 2. A proof of satisfiability

$$
\begin{array}{cl}
(W,1)\colon \Diamond_{\mathsf{Tim}} p_{boy}(bob_{per}) \land \Box_{\mathsf{Tim}}(\forall x_{per})\Diamond_{\mathsf{Sit}}(\neg p_{male}(x_{per})), \blacksquare\Box_{\mathsf{Tim}}(\forall y_{per})(p_{boy}(y_{per}) \to \Box_{\mathsf{Sit}} p_{boy}(y_{per})) & \\ \hline
(W,1)\colon \Diamond_{\mathsf{Tim}} p_{boy}(bob_{per}) \land \Box_{\mathsf{Tim}}(\forall x_{per})\Diamond_{\mathsf{Sit}}(\neg p_{male}(x_{per})), \Box_{\mathsf{Tim}}(\forall y_{per})(p_{boy}(y_{per}) \to \Box_{\mathsf{Sit}} p_{boy}(y_{per})) & (\blacksquare\Box) \\ \hline
(W,1)\colon \Diamond_{\mathsf{Tim}} p_{boy}(bob_{per}), \Box_{\mathsf{Tim}}(\forall x_{per})\Diamond_{\mathsf{Sit}}(\neg p_{male}(x_{per})), \Box_{\mathsf{Tim}}(\forall y_{per})(p_{boy}(y_{per}) \to \Box_{\mathsf{Sit}} p_{boy}(y_{per})) & (\alpha) \\ \hline
(Tim,2)\colon p_{boy}(bob_{per}) \land E(bob_{per}), (\forall x_{per})\Diamond_{\mathsf{Sit}}(\neg p_{male}(x_{per})), (\forall y_{per})(p_{boy}(y_{per}) \to \Box_{\mathsf{Sit}} p_{boy}(y_{per})) & (\pi_{i\mapsto j}) \\ \hline
(Tim,2)\colon p_{boy}(bob^2_{per}), E(bob^2_{per}), (\forall x_{per})\Diamond_{\mathsf{Sit}}(\neg p_{male}(x_{per})), (\forall y_{per})(p_{boy}(y_{per}) \to \Box_{\mathsf{Sit}} p_{boy}(y_{per})) & (\alpha) \\ \hline
(Tim,2)\colon p_{boy}(bob^2_{per}), E(bob^2_{per}), \Diamond_{\mathsf{Sit}}(\neg p_{male}(bob_{per})), (\forall y_{per})(p_{boy}(y_{per}) \to \Box_{\mathsf{Sit}} p_{boy}(y_{per})) & (\gamma_\tau) \\ \hline
(Tim,2)\colon p_{boy}(bob^2_{per}), E(bob^2_{per}), \Diamond_{\mathsf{Sit}}(\neg p_{male}(bob_{per})), p_{boy}(bob_{per}) \to \Box_{\mathsf{Sit}} p_{boy}(bob_{per}) & (\gamma_\tau) \\ \hline
\begin{array}[t]{ll}
(Tim,2)\colon p_{boy}(bob^2_{per}), \neg p_{boy}(bob_{per}) &
\begin{array}[t]{cl}
(Tim,2)\colon p_{boy}(bob^2_{per}), E(bob^2_{per}), \Diamond_{\mathsf{Sit}}(\neg p_{male}(bob_{per})), \Box_{\mathsf{Sit}} p_{boy}(bob_{per}) & \\ \hline
(Sit,3)\colon \neg p_{male}(bob_{per}) \land E(bob_{per}), p_{boy}(bob_{per}) \lor \neg E(bob_{per}) & (\pi_{i\mapsto j}) \\ \hline
(Sit,3)\colon \neg p_{male}(bob_{per}), E(bob^3_{per}), p_{boy}(bob_{per}) \lor \neg E(bob_{per}) & (\alpha) \\ \hline
\begin{array}[t]{ll}
\begin{array}[t]{cl}
(Sit,3)\colon \neg p_{male}(bob_{per}), p_{boy}(bob^3_{per}) & \\ \hline
(Sit,3)\colon \neg p_{male}(bob_{per}), p_{male}(bob^3_{per}) & (<)
\end{array} &
\begin{array}[t]{cl}
(Sit,3)\colon \neg p_{male}(bob_{per}), \neg E(bob_{per}) & \\ \hline
(Sit,3)\colon \bot & (E)
\end{array}
\end{array} & (\beta)
\end{array}
\end{array} & (\beta)
\end{array}
$$

Fig. 3. A proof of satisfiability

Let us prove that the following sorted modal formula is Σ^+-valid by using the calculus.

$$F = p_{apple}(c_{fruit}) \to \blacksquare p_{apple}(c_{fruit})$$

(if c_{fruit} is an apple, then it is an apple in any world as long as it exists) where $T = \{apple,\ fruit,\ \top\}$, $S_A = \emptyset$, $\leq\, = \{(apple,\ fruit)\}$, $C = \{c\}$, $P = \{p_{apple}, p_{fruit},\ p_\top\}$, $\Omega = \{c\colon\ \to fruit,\ p_{apple}\colon fruit,\ p_{fruit}\colon \top,\ p_\top\colon \top\}$, and $apple$ and $fruit$ are inextensible in Σ.

In order to determine the validity of this formula, it is sufficient to check the satisfiability of its negation $\neg F$, i.e., F is Σ^+-valid if and only if $\neg F$ is Σ^+-unsatisfiable. To test the satisfiability of any closed formula, we need to transform the formula into an equivalent one in negation normal form (i.e., negation occurs only in front of an atomic formula). $FV(F)$ denotes the set of free variables occurring in a formula F. Let F_1 and F_2 be formulas where $FV(F_1) \subseteq FV(F_2)$ and $FV(F_1) \cup FV(F_2) = \{x^1_{s_1}, \dots, x^n_{s_n}\}$. $F_1 \simeq F_2$ is a semantic equivalence if for every sorted Σ^+-structure $M = (W, w_0, R, R', U, I)$ and for every $w \in W$, $w \models F_1[x^1_{s_1}/\bar{d}_1, \dots, x^n_{s_n}/\bar{d}_n]$ if and only if $w \models F_2[x^1_{s_1}/\bar{d}_1, \dots, x^n_{s_n}/\bar{d}_n]$.

By the semantic equivalences, the formula $\neg F$ is transformed into an equivalent one in negation normal form as follows:

$$\begin{aligned}\neg(p_{apple}(c_{fruit}) \rightarrow \blacksquare p_{apple}(c_{fruit})) &\simeq p_{apple}(c_{fruit}) \wedge \neg\blacksquare p_{apple}(c_{fruit}) \\ &\simeq p_{apple}(c_{fruit}) \wedge \blacklozenge(\neg p_{apple}(c_{fruit}))\end{aligned}$$

Fig.2 illustrates a proof of testing the satisfiability of the formula $\neg F$ where every tableau for $S = \{\neg F\}$ is closed. This derives that the formula $\neg F$ is Σ^{+}-unsatisfiable, and hence F is Σ^{+}-valid.

Furthermore, consider testing the validity of the following sorted modal formula:

$$F' = \Diamond_{\mathbf{Tim}} p_{boy}(bob_{person}) \rightarrow \Diamond_{\mathbf{Tim}}(\exists x_{person})(\Box_{\mathbf{Sit}} p_{male}(x_{person}))$$

(if Bob is a boy at a time, a person exists at a time who is male in any situation within the time) where $T = \{person,\ male,\ animal,\ \top\}$, $S_A = \{boy\}$, $\leq = \{(boy, person),\ (boy, male),\ (boy, animal),\ (person, animal),\ (male, animal)\}$, $C = \{bob\}$, $P = \{p_{person},\ p_{male},\ p_{animal},\ p_{boy},\ p_{\top}\}$, $\Omega = \{bob\colon\ \rightarrow person,\ p_{person}\colon\ animal,\ p_{male}\colon\ animal,\ p_{animal}\colon\ \top,\ p_{boy}\colon\ person,\ p_{\top}\colon \top\}$, and *boy* is time dependent in Σ. The formula $\neg F'$ is transformed into an equivalent one in negation normal form as follows:

$$\begin{aligned}&\neg(\Diamond_{\mathbf{Tim}} p_{boy}(bob_{person}) \rightarrow \Diamond_{\mathbf{Tim}}(\exists x_{person})(\Box_{\mathbf{Sit}} p_{male}(x_{person}))) \\ &\simeq \Diamond_{\mathbf{Tim}} p_{boy}(bob_{person}) \wedge \neg\Diamond_{\mathbf{Tim}}(\exists x_{person})(\Box_{\mathbf{Sit}} p_{male}(x_{person})) \\ &\simeq \Diamond_{\mathbf{Tim}} p_{boy}(bob_{person}) \wedge \Box_{\mathbf{Tim}}\neg(\exists x_{person})(\Box_{\mathbf{Sit}} p_{male}(x_{person})) \\ &\simeq \Diamond_{\mathbf{Tim}} p_{boy}(bob_{person}) \wedge \Box_{\mathbf{Tim}}(\forall x_{person})(\neg\Box_{\mathbf{Sit}} p_{male}(x_{person})) \\ &\simeq \Diamond_{\mathbf{Tim}} p_{boy}(bob_{person}) \wedge \Box_{\mathbf{Tim}}(\forall x_{person})\Diamond_{\mathbf{Sit}}(\neg p_{male}(x_{person}))\end{aligned}$$

In Fig.3, we show a proof of testing the satisfiability of the formula $\neg F'$. Since every tableau for $S' = \{\neg F'\}$ is closed, F' is Σ^{+}-valid.

5 Conclusion

The main results of this paper are (i) a refinement of the ontological property classification by means of individual existence and time and situation dependencies and (ii) an integration of sort predicates and sorted terms (in order-sorted logic), modalities and varying domains (in quantified modal logic), and temporal operators (in first-order temporal logic) in order to model the ontological distinctions among properties.

We formalized the syntax, semantics, and inference system (with axioms) for an order-sorted modal logic such that they are well-suited to deal with the ontological notions. The formal semantics of properties is practically and theoretically useful in deciding the *ontological* and philosophical suitability of property descriptions in information systems and for guaranteeing *logical* consistency in reasoning about properties. We presented a prefixed tableau calculus by extending Meyer and Cerrito's calculus. To handle the multi-modal operators with individual existence, our calculus derives existential predicate formulas and processes formulas prefixed by a pair (i, n) of the kind of worlds i and a natural number n (as a world).

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